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METHOD FOR PREDICTING THE STIFFNESS OF WOOD-JOIST FLOOR SYSTEMS WITH PARTIAL COMPOSITE ACTION

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ABSTRACT

Residential wood floor systems have long been designed by considering the joists to be simple beams which act independently in supporting the imposed loads. However, interaction between the joists and sheathing material increases the stiffness above that of the joists alone. The interaction is not complete, however, due to the nonrigid behavior of the mechanical or adhesive fasteners which attach the sheathing to the joists. Gaps in the sheathing disrupt its continuity and further complicate the analysis.

By use of methods for computing the stiffness of composite beams and for predicting the load-slip characteristics of individual mechanical fasteners, complex computational procedures were reworked and combined into an easy-to-use format. The problem of open gaps in the sheathing was handled by a simple modification of the basic method. This procedure gave excellent correlation between computed and experimental values obtained at FPL and elsewhere.

With known material properties and the procedures presented, it is possible for designers to easily and accurately predict floor stiffness properties.

METHOD FOR PREDICTING THE STIFFNESS OF WOOD-JOIST FLOOR SYSTEMS WITH PARTIAL COMPOSITE ACTION

By

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Introduction

Residential wood floor systems have long been designed by considering the joists to be simple beams which act independently of each other and of the other materials which compose the floor. This simplified procedure, despite its amost universal acceptance and use, neglects many factors that affect the strength, rigidity, and performance of a floor. It is generally recognized that this results in floor systems which are stiffer than assumed. Therefore, more accurate methods of analysis and design can result in floors which are structurally sound, but which can be built more efficiently than those which are designed by present methods.

Interaction between the joists and sheathing material increases the stiffness and strength above those of the joists alone. These increases have been investigated by several researchers.

In tests performed at the Forest Products Laboratory[2] the additional stiffness due to diagonal board subflooring and oak finish flooring was only 10 percent. Although Russell (9)[3] concluded from this that "the traditional custom of neglecting the subfloor and finish floor, and designing on the basis of the joists alone, appears to be sound practice," more recent studies which investigated more modern construction techniques have shown much greater inprovements in performance (7).

In a series of tests run by the National Association of Home Builders (6), a 13 percent stiffness increase was recorded with a nailed plywood subfloor, and the increase was 38 percent when the plywood was nail-glued to the joists. Similarly, Williston and Abner (16) reported that complete floor systems deflected an average of 40 percent less than the joists alone, and Hurst (1) noted substantial decreases in deflection due to composite action and other factors. Considerable stiffness increases have also been reported in recent studies by Polensek et al. (8) and by Vanderbilt et al. (10, 11).

The transverse (i.e., perpendicular to joist span) stiffness of subflooring tends to reduce differences in the joist deflections when a floor is subjected to uniform loading (8, 10, 11). Thus, this "two-way" action diminishes the effects of variation in joist stiffness, and tends to justify the practice of designing on the basis of equal stiffness for all the joists.

Kuenzi and Wilkinson (2) have presented a method for computing the stiffness of composite beams, and Wilkinson's subsequent research (12, 13, 14, 15) on predicting the behavior of connections made with mechanical fasteners permits the development of a more rigorous procedure for computing the stiffness of floors with joist-subfloor interaction.

Clearly, interaction between the joists and sheathing material increases the stiffness of a wood-joist floor system above that of the joists alone. The interaction is not complete, however, due to the non-rigid behavior of the mechanical or adhesive fasteners which attach the subfloor to the joists. Also, gaps in the sheathing disrupt its continuity and further complicate the analysis.

Objective and Scope

The objective of this paper is to present a procedure by which the deflection of wood-joist floors may be computed. The derivation is limited to the case of two-layer floors and considers the effects of fastener stiffness and noncontinuous sheathing.

Theoretical Development

Assumptions

In developing the procedure, several assumptions and limitations are imposed. The floors to be considered consist of two layers: the joists and a single layer of sheathing. They are single-span with simple supports. Loads and deflections in the transverse direction are assumed to be constant (i.e., all joists have the same bending stiffness). Thus, the basic structural unit under consideration is a simple T-beam consisting of the joist web and subfloor flange (fig.1). It will be assumed that the width of the flange is equal to the center - to - center spacing of the joists. This assumption has been found to be approximately valid for reasonable spacings, and even moderate reductions in effective width would not greatly affect the results of the analysis (11). When gaps in the sheathing are considered, it is assumed that they are evenly spaced along the span of the joist. Similarly, mechanical fasteners are assumed to be evenly spaced and therefore produce a uniform distribution of fastener rigidity along the length of the beam.
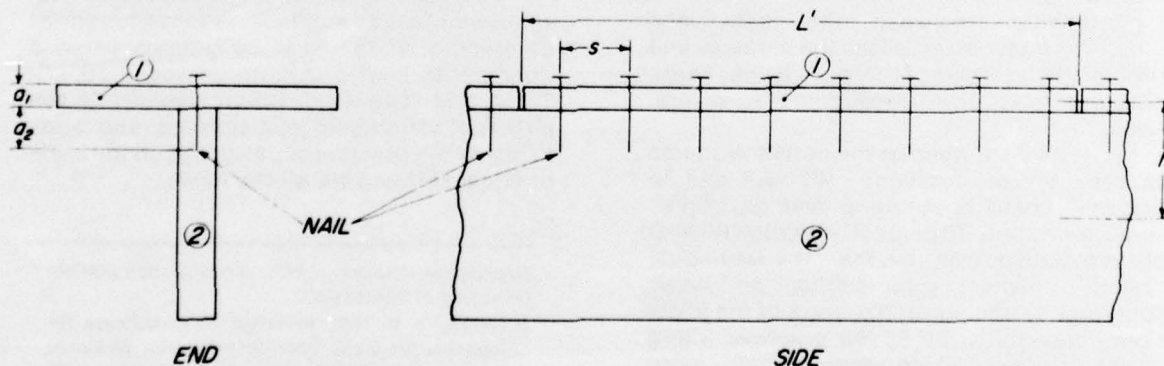

Figure 1. -- T-beam with subfloor flange (1) and joist web (2).

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Nailed Connections

Wilkinson (14) has shown how the behavior of nailed two-member joints can be defined in terms of the physical dimensions and properties of the joint's components. The slope of the load/slip curve can be expressed in one of three forms, depending upon the value of a characteristic, λ, for each member, which is defined as:

$$\lambda = \sqrt[4]{\frac{k_o d_N}{4 E_N I_N}} \qquad (1)$$

where k_o = elastic bearing constant for the material (lb/in.³),

d_N = nail diameter (in.), and

$E_N I_N$ = bending stiffness of the nail (lb-in.²)

Denoting the nail penetration into each of the two members as a_1 and a_2 (see fig. 1), and the characteristic values for each member as λ_1 and λ_2, the load/slip equation for the joint takes one of three forms, depending upon whether $\lambda_1 a_1$ and $\lambda_2 a_2$ are both less than 2, both greater than 2, or if one is less than and the other greater than 2.

Values of the elastic bearing constant, k_o, for various materials have been determined (13, 15) and those of greatest interest are summarized in table 1. Because λ is proportional to the fourth root of the bearing constant, k_o (Eq. (1)), computation of λ is fairly insensitive to even moderate variations in the value of k_o. For example, a 25 percent increase in the value of k_o will increase λ by less than 6 percent, and even doubling k_o will increase λ by only 19 percent. Similarly, it has been shown that the amount of composite action is insensitive to moderate changes in the stiffness of the connector system (4, 11). Therefore, only approximate values for k_o are necessary in computing the load/slip values for nailed connections. The following values have been chosen as representative of the materials which usually compose a wood-joist floor system (see table 1):

k_o (Plywood or hardboard subfloor sheathing)

$$= 0.8 \times 10^6 \; lb/in.^3$$

k_o (wood joists, s.g. ≈ 0.45)

$$= 1.0 \times 10^6 \; lb/in.^3 \qquad (2)$$

Using these values of k_o and the aforementioned load/slip equations (14), the values of table 2 were computed. The penetration of the nail in sheathing, a_1, is equal to the sheathing thickness, and the penetration into the joist, a_2, is equal to the nail length minus a_1. Sample calculations are shown in Appendix A.

The shear/slip per unit length, S (lb/in.²), for a series of evenly spaced nails is required for the composite beam analysis. This can be computed by simply dividing the load/slip (P/δ) for a single nail (as determined from table 2) by the nail spacing, s (in.):

$$S = \frac{P/\delta}{s} \qquad (3)$$

Table 1. -- Elastic bearing constants for smooth nails

Material	Bearing constant, k_o[1]
	10⁶ lb/in.³
Solid wood, loaded parallel to grain	2.144 g
Douglas-fir plywood	0.800
Wet-formed hardboard	0.950 g

[1] g is the specific gravity of the material.

Table 2. -- Load/slip (lbs/in.) for common wire nails[1]

Nail size	Sheathing thickness, a_1 (in.)					
	1/2	5/8	3/4	7/8	1	1-1/8
2d	3,600	3,300	2,900	2,800	--	--
4d	5,700	6,100	6,100	6,100	5,900	5,200
5d	5,700	6,100	6,100	6,100	6,100	6,100
6d, 7d	6,900	7,800	7,900	7,900	7,900	7,900
8d, 9d	8,600	9,400	10,200	10,200	10,200	10,200
10d, 12d	10,500	11,200	12,300	12,600	12,600	12,600
16d	12,100	12,800	13,900	14,800	14,800	14,800
20d	16,100	16,600	17,600	18,900	19,900	19,900
30d	18,300	18,700	19,600	21,000	22,600	22,700
40d	21,200	21,400	22,200	23,500	25,200	26,200
60d	27,800	27,900	28,400	29,600	31,200	33,100

[1]k_{O_1} (sheathing) $= 0.8 \times 10^6$ lbs/in.^{3}

k_{O_2} (joist) $= 1.0 \times 10^6$ lbs/in.^{3}

E (steel nail) $= 30.0 \times 10^6$ lb/in.^{2}

Adhesive Connections

The shear/slip per unit length, S, of an adhesive joint is computed by multiplying the adhesive's shear modulus, G (lb/in.^{2}), by the width of the joint, b (in.), and dividing by the thickness of the glueline, t (in.):

$$S = \frac{Gb}{t} \qquad (4)$$

The shear modulus of construction adhesives varies greatly, and no single typical value can be selected, as was the case for the properties relating to nailed connections. Similarly, the thickness is variable, usually around 1/32 inch. The width b is usually equal to the width of the joist (1.5 in. for dimension lumber).

Analysis of Composite Action

Equations have been developed which define the deflections of simple beams with partial composite action (2). Individual equations derived for the three load cases considered (midspan and quarter-point concentrated loads and distributed load) can all be written in the same general form:

$$\Delta = \Delta_R \left[1 + f_\Delta \left(\frac{EI_R}{EI_U} - 1 \right) \right] \qquad (5)$$

where Δ = deflection of the beam,

Δ_R = deflection if the components of the beam are rigidly connected[4],

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EI_R = bending stiffness if the components are rigidly connected,

EI_U = stiffness if the components are completely unconnected,

$f_\triangle$ = a constant involving hyperbolic trigonometric functions of $L\alpha$,

L = beam span

$$\alpha^2 = \frac{h^2 S}{EI_R - EI_U}\left(\frac{EI_R}{EI_U}\right) \qquad (6)$$

h = distance between the centroidal axes of the joist and sheathing (fig. 1)

S = load per unit length which causes a unit slip in the nail or adhesive joint

The factor $f_\triangle$ contains hyperbolic trigonometric functions of $L\alpha$ and its exact form depends upon the type of loading (2). In attempting to find a simple method for computing $f_\triangle$ it was found that for all three load cases mentioned above, $f_\triangle$ can be closely approximated by:

$$f_\triangle = \frac{10}{(L\alpha)^2 + 10} \qquad (7)$$

A comparison of approximate values and exact values of $f_\triangle$ is presented in table 3. The footnote to the table also lists the exact equations for $f_\triangle$. As can be seen, the exact and approximate values are in close agreement.

The bending stiffnesses EI_U and EI_R, which are required in Eq. (5) can be computed easily. EI_U is merely the summation of the individual stiffnesses of the joist and the sheathing. Usually, the subfloor stiffness is very small in comparison to the joist stiffness and can be neglected in the computation. The

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⁵For example, for a uniform load, w(lb/unit length),

$$\triangle_R = \frac{5}{384} \frac{wL^4}{EI_R}$$

value EI_R can be computed for a rigidly connected T-beam by:

$$EI_R = EI_U + \frac{(EA_1)(EA_2)}{EA_1 + EA_2} h^2 \qquad (8)$$

where EA_1 and EA_2 = the axial stiffnesses of the flange and web, respectively, and, h = the distance between the centroids of the flange and web (fig. 1).

A derivation of Eq. (8) is presented in Appendix B.

The method for computing the deflection of a wood joist floor with constant properties is now complete. First, the stiffness values EI_U and EI_R are computed from the properties of the joist and sheathing (Eq. (8)) and the fastener stiffness is computed from table 2 and Eq. (3) for common nails, or from Eq. (4) for adhesive. Then α is calculated from Eq. (6) and inserted in Eq. (7) to give $f_\triangle$. Finally, Eq. (5) gives the deflection.

Since it is common in the literature to speak of stiffness increases (above EI_U) due to partial composite action or of stiffness decreases (below EI_R) due to incomplete interaction, the effective stiffness, EI, of the joist-subfloor assembly can be expressed in terms of EI_U or EI_R. Noting that $\triangle / \triangle_R = EI_R / EI$,

Eq. 5 can be rewritten as:

$$EI = \frac{EI_R}{1 + f_\triangle\left(\frac{EI_R}{EI_U} - 1\right)} \qquad (9)$$

or

$$EI = \frac{EI_U}{\frac{EI_U}{EI_R} + f_\triangle\left(1 - \frac{EI_U}{EI_R}\right)}$$

$$= \frac{EI_U}{\frac{EI_U}{EI_R}\left(1 - f_\triangle\right) + f_\triangle} \qquad (10)$$

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Table 3. -- Comparison of approximate and exact values of $f_\triangle$

Lα	Approximate $f_\triangle$[1]	Exact $f_\triangle$		
		Quarter-point loading[2]	Distributed loading[3]	Midspan loading[4]
0.0	1.000	1.000	1.000	1.000
0.5	.976	.975	.975	.976
1.0	.909	.907	.908	.909
1.5	.816	.812	.814	.817
2.0	.714	.708	.711	.715
2.5	.615	.608	.611	.617
3.0	.526	.518	.522	.529
3.5	.449	.440	.445	.453
4.0	.385	.375	.380	.388
5.0	.286	.276	.281	.291
6.0	.217	.208	.213	.223
7.0	.169	.161	.166	.175
8.0	.135	.127	.132	.141
9.0	.110	.103	.107	.115
10.0	.091	.084	.088	.096
15.0	.043	.039	.041	.046
20.0	.024	.022	.024	.027
30.0	.011	.010	.011	.012
50.0	.004	.003	.004	.005
100.0	.001	.001	.001	.001
∞	0.	0.	0.	0.

$$[1]f_\triangle = \frac{10}{(L\alpha)^2 + 10}$$

$$[2]f_\triangle = \frac{24}{11}\left(\frac{2}{L\alpha}\right)^2\left(1 - \frac{sinh\frac{L\alpha}{4}}{\frac{L\alpha}{4}cosh\frac{L\alpha}{2}}\right)$$

$$[3]f_\triangle = \frac{12}{5}\left(\frac{2}{L\alpha}\right)^2\left[1 - 2\left(\frac{2}{L\alpha}\right)^2\left(1 - \frac{1}{cosh\frac{L\alpha}{2}}\right)\right]$$

$$[4]f_\triangle = 3\left(\frac{2}{L\alpha}\right)^2\left(1 - \frac{tanh\frac{L\alpha}{2}}{\frac{L\alpha}{2}}\right)$$

When the connection is completely rigid ($S = \infty$), $f_\Delta = 0$ and $EI = EI_R$. When there is no connection ($S = 0$), $f_\Delta = 1$ and $EI = EI_U$. When the effective stiffness, EI, is computed, the beam deflection can be computed directly by the elementary beam equation, rather than by Eq. (5).

Gaps in the Sheathing

Thus far, the analysis of the composite action has not considered the effect of gaps in the sheathing which forms the flange of the T-beam. Such gaps disrupt the continuity of the flange and can drastically reduce the amount of composite action. An examination of the preceding formulation suggests a method by which this important effect might easily be included.

The factor f_Δ, which defines the amount of composite action, is computed by Eq. (7). Its value depends upon the values of α and L. The value of α is entirely dependent upon the cross-sectional properties and dimensions of the construction (Eq. (6)). Thus, for two beams which are identical except for span, L, the one with the shorter span will exhibit a lower apparent stiffness because f_Δ will be larger. As the span is shortened, f_Δ approaches

unity and EI approaches EI_U. This is analogous to what would happen if gaps were cut into an originally continuous flange. When the flange is continuous for the full span, f_Δ would have the value computed by Eq. (7). As gaps were inserted, the composite action would be disrupted and f_Δ would increase toward unity.

Thus, it would appear that the problem of gaps can be accommodated by modifying the factor L in the denominator of Eq. (7). This is done by rewriting Eq. (7) as:

$$f_\Delta = \frac{10}{(L'\alpha)^2 + 10} \qquad (11)$$

where $L' =$ distance between the open gaps in the sheathing (fig. 1).

It is assumed that the gaps are evenly spaced across the span. For a continuous flange with no gaps, $L' = L$ and Eq. (11) is identical to Eq. (7). As the number of gaps increases, L' approaches zero and f_Δ approaches unity.

It is seen in the following section that this modification works well when comparing computed values to experimental data.

Experimental Floor Evaluations

Construction

Seven floors were constructed at the Forest Products Laboratory. Each consisted of nine No. 2 and better 2 - by - 8 Douglas-fir joists spaced 16 inches on centers. Floors were 13 feet 4 inches wide and were tested on a span of 12 feet.

Four of the floors (designated N-1 to N-4) used standard grade 5/8-inch plywood nailed to the joists with the face grain perpendicular to the span. Nails were 8d at 6 inches along the edges and 8 inches over intermediate joists. A 1/16-inch gap was left between the plywood sheets. The other three floors (G-1 to G-3) had the plywood nail-glued in place using a rigid adhesive. These used tongue-and-groove plywood with the edges glued to give a continuous subfloor. The joist stiffnesses were determined via nondestructive static bending tests before the floors were assembled.

Evaluation

The floors were subjected to both concentrated and uniform load tests. The concentrated point loads were applied by means of a hydraulic jack placed over the center joist at midspan (fig. 2). Uniform loads were applied with an air bag (fig. 3) and were carried to destruction. In all tests, midspan joist deflections were measured by means of linear resistance potentiometers mounted on yokes supported at the neutral axes of the joists over the simple supports (fig. 4).

The concentrated load was twice applied in increments of 100 pounds to a total load of 500 pounds. After the removal of the hydraulic jack and the placement of the air bag for the uniform load tests, each floor was loaded three times to 40 pounds per square foot in increments of 10 pounds per square foot. On the fourth and final run, the test was carried to failure.

Results

The results of the tests are summarized in table 4. The deflection at 50 pounds per square foot uniform load represents the average deflection of the center five joists. This is to eliminate the edge effects of the end joists. The "failure load" is the uniform load which produced the first joist fracture.
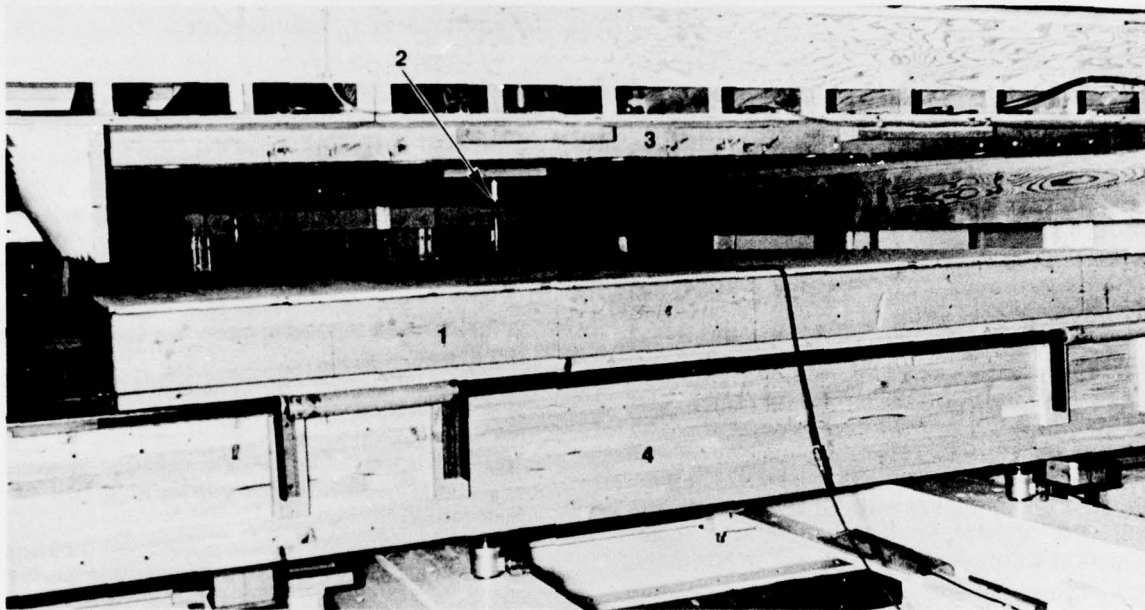

Figure 2. -- Concentrated load test showing floor specimen (1), hydraulic jack and load cell (2), strongback (3), and support (4). (M 139 508-9)
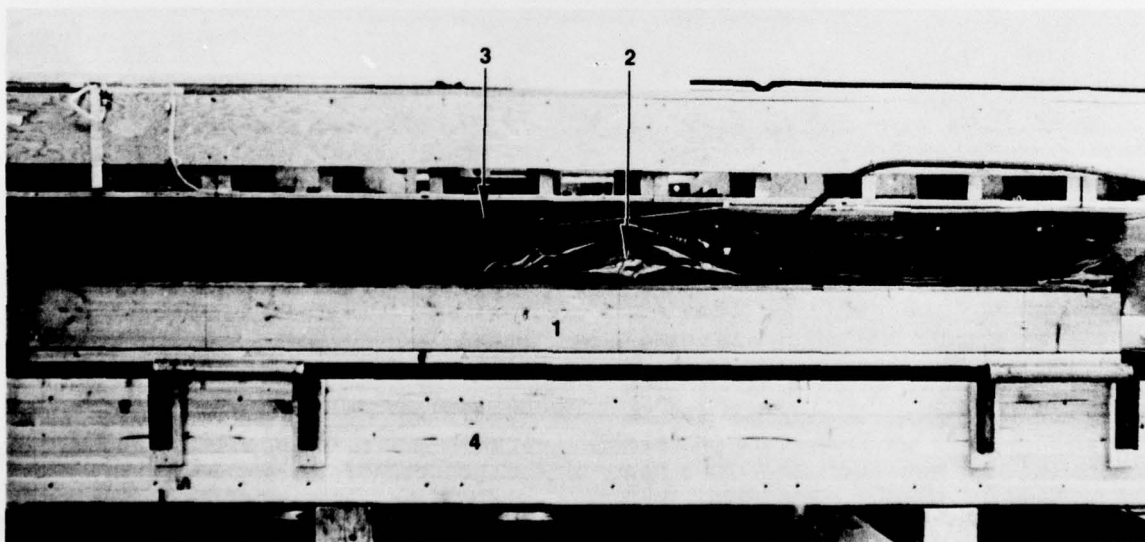

Figure 3. -- Distributed load set-up showing floor (1), air bag (2), strongback (3), and support (4). During test, floor was raised to reduce gap between the test specimen and the strongback. (M 139 508-12)

Table 4. -- Summary of floor tests

Floor	Average E	Deflection at 500 pounds concentrated load[1]	Deflection at 50 psf uniform load[2]	Failure load
	10^6 lbs/in.^{2}	In.	In.	Lb/ft^2
Sheathing nailed				
N-1	1.57	0.155	0.376	128
N-2	2.20	.126	.280	190
N-3	2.00	.115	.312	120
N-4	1.49	.152	.368	140
Sheathing rigidly glued				
G-1	1.53	.104	.246	200
G-2	2.20	.087	.189	315
G-3	2.00	.095	.190	225

[1]Deflection at midspan of loaded joist.
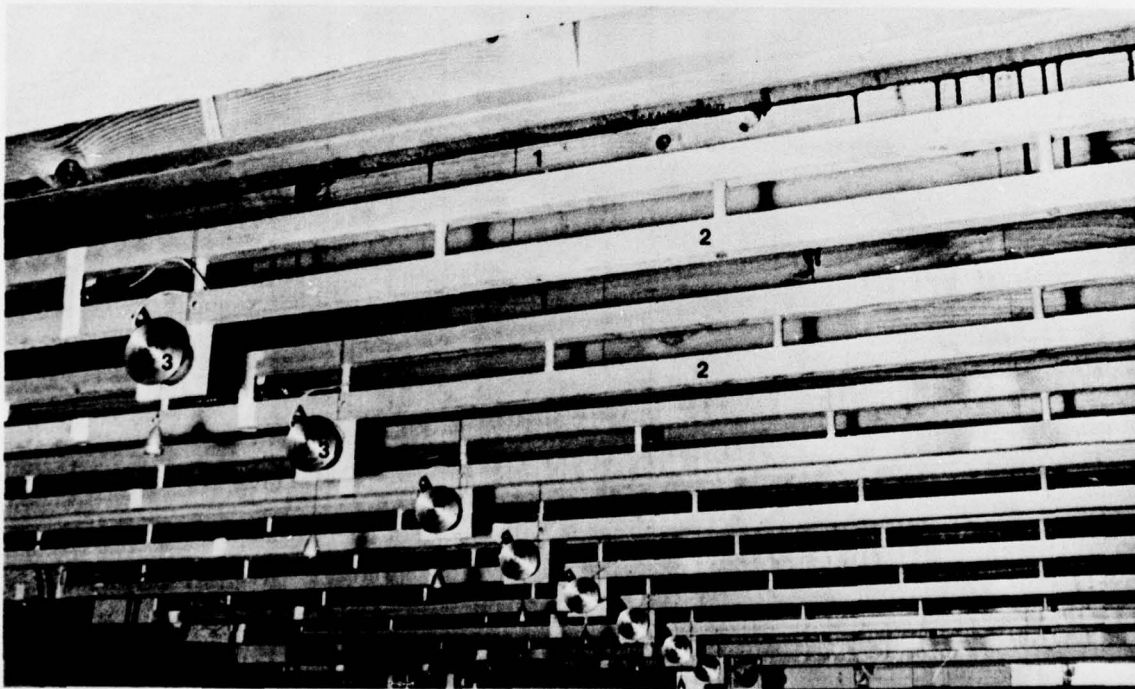[2]Average deflection of center five joists.

Figure 4. -- Underside of test floor showing joists (1), yokes (2), and potentiometers (3).

(M 139 508-17)

Comparison of Theoretical and Experimental Results

The theoretical method developed in this study was used to calculate the deflections of experimental floors and T-beams tested at the Forest Products Laboratory as described above, at Colorado State University (CSU) (11), and by the NAHB Research Foundation (5).

Table 5 presents a comparison of computed and experimental deflections for the FPL floor tests and the CSU T-beams. The agreement is very good: 22 of the 29 computed deflection (76 pct) are within 5 percent of the observed experimental values. The CSU data are for two-joist T-beams at various midspan concentrated loads. In the CSU study (11), distinction was made between "flexible" and "open" gaps, where the flexible type is capable of transmitting some force and the open type transmits none. Only evenly spaced open gaps were considered in this comparison. Sample calculations for the uniformly loaded FPL floors and a CSU T-beam are given in Appendix C.

Table 5. -- Comparison of experimental and theoretical deflections

	Specimen number	Joist number	Observed deflection	Computed deflection	Ratio
			In.	In.	
FPL floors[1]	N-1		0.376	0.394	1.05
	N-2		.280	.285	1.02
	N-3		.312	.313	1.00
	N-4		.368	.415	1.13
	G-1		.246	.235	.96
	G-2		.189	.184	.97
	G-3		.190	.197	1.04
CSU T-beams[2]	T4	1	.262	.258	.98
		2	.353	.363	1.03
	T5	1	.338	.336	.99
		2	.354	.349	.99
	T8	1	.399	.370	.93
		2	.386	.380	.98
	T9	1	.336	.317	.94
		2	.340	.287	.84
	T10 (2 gaps)	1	.252	.263	1.04
		2	.242	.265	1.10
	(5 gaps)	1	.259	.271	1.05
		2	.253	.274	1.08
	T12 (1 gap)	1	.340	.323	.95
		2	.325	.322	.99
	(5 gaps)	1	.391	.379	.97
		2	.386	.378	.98
	T13	1	.307	.321	1.05
		2	.346	.389	1.12
	T14	1	.184	.181	.98
		2	.205	.199	.97
	T15	1	.327	.311	.95
		2	.332	.320	.96

Average...1.002

Standard deviation...0.061

[1]At 50 lb/ft² (average of 5 joists).

[2]With open gaps at various concentrated loads.

The test data presented in the NAHB Research Foundation Report (5) give values of EI_U, EI_R, and experimental EI for 11 different floors with nail-glued plywood subfloors. However, values for the stiffness of the glue joints are not presented. Therefore, it is not possible to perform the same sort of direct computations as for the FPL and CSU data. It is possible, however, to select trial values for S and compute the corresponding values for effective EI. The results of such calculations are shown in table 6 for S = 25-, 50-, and 100-thousand pounds per square inch. For S = 50,000, agreement is good between the experimental and calculated stiffnesses. Such a value for S is quite reasonable for a high-quality adhesive joint produced under laboratory conditions.

Table 6. -- Comparison of NAHB (5) test results and calculated stiffness for assumed values of S

Floor number	NAHB stiffnesses (5)			Calculated stiffnesses					
	EI	EI_R	EI (test)	S = 25,000 lb/in.^{2}		S = 50,000 lb/in.^{2}		S = 100,000 lb/in.^{2}	
				EI	Ratio	EI	Ratio	EI	Ratio
	10^6 lb-in.^{2}	10^6 lb-in.^{2}	10^6 lb-in.^{2}	10^6 lb-in.^{2}		10^6 lb-in.^{2}		10^6 lb-in.^{2}	
1	24.82	65.69	47.59	47.67	1.00	54.13	1.14	58.96	1.24
2	37.41	84.65	66.61	62.13	.93	69.86	1.05	75.88	1.14
3	24.83	70.99	57.01	50.18	.88	57.56	1.01	63.13	1.11
4	37.46	92.49	76.31	65.28	.86	74.41	.98	81.68	1.07
5	263.1	484.11	409.61	367.8	.90	405.2	.99	436.0	1.06
6	327.8	569.92	514.98	437.0	.85	478.3	.93	513.4	1.00
7	266.1	521.40	472.44	380.5	.81	424.1	.90	461.3	.98
8	328.4	595.57	580.92	445.1	.77	490.9	.85	530.5	.91
9	23.56	55.84	41.80	43.45	1.04	48.18	1.15	51.49	1.23
10	186.9	384.43	281.17	269.0	.96	302.9	1.08	333.0	1.18
11	262.1	620.95	546.68	393.5	.72	454.5	.83	512.6	.94
Average ratio (EI_{calc} /EI_{test})				0.882		0.990		1.078	
Standard deviation				0.097		0.109		0.114	

Summary and Conclusions

This paper has presented a simplified method for computing the deflections of wood-joist floor systems. The composite stiffness can be calculated from the properties of the individual components by means of a series of simple equations. Gaps in the sheathing can be accommodated by a slight modification of the basic method. Computations performed by this method compare closely with results obtained experimentally.

It has been assumed that the loads and deflections are constant in the transverse direction. However, it is expected that this procedure will also work well for floors subjected to concentrated loads, once the distribution of the load to the individual joists has been determined by other means.

Literature Cited

1. Hurst, H. T.
   1965. The wood frame house as a structural unit, Part I Floor deflection as influenced by various stages of construction, Va. Agric. Exp. Stat., VPI Tech. Bull. 179. Va. Polytechnic Institute and State Univ., Blacksburg, Va.

2. Kuenzi, E. W., and T. L. Wilkinson.
   1971. Composite beams -- Effect of adhesive or fastener rigidity. USDA For. Serv. Res. Pap. FPL 152. For. Prod. Lab., Madison, Wis.

3. Kuo, M. L., M. E. Criswell, J. R. Goodman, J. Bodig, E. G. Thompson, and M. D. Vanderbilt.
   1974. Verification of a mathematical model for layered T-beams. Str. Res. Rep. No. 10, Civil Eng. Dept., Colo. State Univ., Fort Collins, Colo.

4. McGee, W. D., and R. J. Hoyle, Jr.
   1974. Design method for elastomeric adhesive bonded wood joist-deck systems. Wood and Fiber 6(2): 144 - 155. Summer.

5. National Association of Home Builders' Research Foundation.
   1973. Performance of glued single-layer plywood-to-wood joist floor systems (vol. 3). Report CR-484. NAHB, Washington, D.C. June.

6. National Association of Home Builders' Research Institute Laboratory.
   1961. The effect of framing and subfloor attachment on the stiffness of residential floors. Report LR-4. NAHB, Washington, D.C.

7. Onysko, D. M.
   1970. Performance of wood-joist floor systems: A literature survey. Inf. Rep. OP-X-24, Eastern For. Prod. Lab., Ottawa, Canada.

8. Polensek, A., G. H. Atherton, S. E. Corder, and J. L. Jenkins.
   1972. Response of nailed wood-joist floors to static loads. For. Prod. J. 22(9): 52 - 61. Sept.

9. Russell, W. A.
   1954. Deflection characteristics of residential wood-joist floor systems. Housing Res. Pap. No. 30, Div. of Housing Res., Housing and Home Finance Agency, Washington, D.C.

10. Vanderbilt, M. D., J. R. Goodman, and M. E. Criswell.
    1974. Service and overload behavior of wood joist floor systems. J. Str. Div., Proc. Amer. Soc. Civil Eng., Vol. 100, No. ST1, P. 11 - 29. Proc. Pap. 10274, Jan.

11. Vanderbilt, M. D., J. R. Goodman, M. E. Criswell, and J. Bodig.
    1974. Final report to the National Science Foundation for Grant GK-30853: A rational analysis and design procedure for wood joist floor systems. Colo. State Univ., Fort Collins, Colo., Nov.

12. Wilkinson, T. L.
    1971. Theoretical lateral resistance of nailed joints. J. Str. Div., Amer. Soc. Civil Eng., vol. 97, No. ST5, p. 1381-1398. Proc. Pap. 8121, May.

13. Wilkinson, T. L.
    1972. Effect of deformed shanks, prebored lead holes, and grain orientation on the elastic bearing constant for laterally loaded nail joints. USDA For. Serv. Res. Pap. FPL 192. For. Prod. Lab., Madison, Wis.

14. Wilkinson, T. L.
    1972. Analysis of nailed joints with dissimilar members. J. Str. Div., Amer. Soc. Civil Eng., Vol. 98, No. ST9, p. 2005-2013. Proc. Pap. 9189, Sept.

15. Wilkinson, T. L.
    1974. Elastic bearing constants for sheathing materials. USDA For. Serv. Res. Pap. FPL 224. For. Prod. Lab., Madison, Wis.

16. Williston, E. M., and T. L. Abner.
    1962. Design criteria for wood floor systems. For. Prod. J. 12(9): 403 - 409. Sept.

Appendix A. -- Calculation of Nail P/δ Values of Table 2

Wilkinson (14) derived equations for computing load/slip, P/δ, for three separate cases: (I) $\lambda_1 a_1 > 2$ and $\lambda_2 a_2 > 2$, (II) $\lambda_1 a_1 < 2$, and $\lambda_2 a_2 > 2$, and (III) $\lambda_1 a_1 < 2$ and $\lambda_2 a_2 < 2$.

Case III does not occur in table 2. The following computations for connections made with 4d nails will demonstrate the use of the equations for cases I and II. In the computations, the following values are used:

k_{o_1} (sheathing)	$= 0.8 \times 10^6 \, lb/in.^3$
k_{o_2} (joist)	$= 1.0 \times 10^6 \, lb/in.^3$
E_N (nail)	$= 30 \times 10^6 \, lb/in.^2$
d_N (4d)	$= 0.098 \, in.$
L_N (nail length)	$= 1.5 \, in.$

For a circular cross-section $I = \frac{1}{64} \pi d^4$ and Eq. (1) can be rewritten as:

$$\lambda = 2 \sqrt[4]{\frac{k_o}{\pi E_N d_N^3}} \qquad (A1)$$

Inserting the above values of k_o, E_N, and d_N gives:

$$\lambda_1 = 3.47 \, in.^{-1}$$
$$\lambda_2 = 3.66 \, in.^{-1} \qquad (A2)$$

For 1/2-inch sheathing:

$$a_1 = 0.5 \, in. \qquad\qquad \lambda_1 a_1 = 1.73$$
$$a_2 = L_N - a_1 = 1.5 - 0.5 = 1.0 \, in. \qquad \lambda_2 a_2 = 3.66$$

Thus, 1/2-inch sheathing is covered by case II, for which

$$r = \frac{k_{o_2}}{k_{o_1}} = 1.25$$

$$\gamma = \lambda_2 a_1 = 1.83$$

$$\beta_2 = \frac{r(3r + \gamma^3)}{2(2r + \gamma)(3r + \gamma^3) - (3r - \gamma^2)^2} = 0.145$$

$$P/\delta = a_1 d_N k_{o_1} \beta_2 = 5700 \, lb/in.$$

For 3/4-inch sheathing:

$$a_1 = 0.75 \, in. \qquad\qquad \lambda_1 a_1 = 2.60$$
$$a_2 = 1.5 - 0.75 = 0.75 \, in. \qquad \lambda_2 a_2 = 2.75$$

So, 3/4-inch sheathing is covered by case I, for which

$$r = \frac{k_{o_2}}{k_{o_1}} = 1.25$$

$$\beta_1 = \frac{r(r + r^{.75})}{2(r + r^{.25})(r + r^{.75}) - (r - r^{.75})^2} = 0.271$$

$$P/\delta = \sqrt{2} \, E_N^{.25} \, I_N^{.25} \, k_{o_1}^{.75} \, d_N^{.75} \, \beta_1 = 6100 \, lb/in.$$

For 1-inch sheathing:

$$a_1 = 1.0 \, in. \qquad\qquad \lambda_1 a_1 = 3.47$$
$$a_2 = 1.5 - 1.0 = 0.5 \, in. \qquad \lambda_2 a_2 = 1.83$$

Thus, 1-inch sheathing is covered by case II, even though the subscripts 1 and 2 are interchanged

$$r = \frac{k_{o_1}}{k_{o_2}} = 0.8$$

$$\beta_2 = \frac{r(3r + \gamma^3)}{2(2r + \gamma)(3r + \gamma^3) - (3r - \gamma^2)^2} = 0.121$$

$$\gamma = \lambda_1 a_2 = 1.73$$

$$P/\delta = a_2 d_N k_{o_2} \beta_2 = 5900 \ lb/in.$$

Appendix B. -- Derivation of Equation (8)

Equation (8) for computing EI_R, the bending stiffness of a T-beam when the components are rigidly connected, can be derived by the "transformed area" method (fig. B-1). Selecting a base E of unity, the transformed areas, $\overline{A}$, and moments of inertia, $\overline{I}$ are:

for the flange: and for the web:

$$\overline{A}_1 = E_1 A_1 \qquad\qquad \overline{A}_2 = E_2 A_2$$

$$\overline{I}_1 = E_1 I_1 \qquad\qquad \overline{I}_2 = E_2 I_2 \qquad \text{(B1)}$$

First, the centroid is located:

$$\overline{y} = \frac{h\overline{A}_1}{\overline{A}_1 + \overline{A}_2} \quad and \quad h - \overline{y} = \frac{h\overline{A}_2}{\overline{A}_1 + \overline{A}_2} \quad \text{(B2)}$$

The composite moment of inertia is:

$$\overline{I} = \overline{I}_1 + (h - \overline{y})^2 \overline{A}_1 + \overline{I}_2 + \overline{y}^2 \overline{A}_2 \qquad \text{(B3)}$$

Substituting equations (B2) into (B3) gives:

$$\overline{I} = \overline{I}_1 + \overline{I}_2 + \frac{\overline{A}_1 \overline{A}_2}{\overline{A}_1 + \overline{A}_2} h^2 \qquad \text{(B4)}$$

This is identical to equation (8) when the relationships of (B1) are re-substitued.
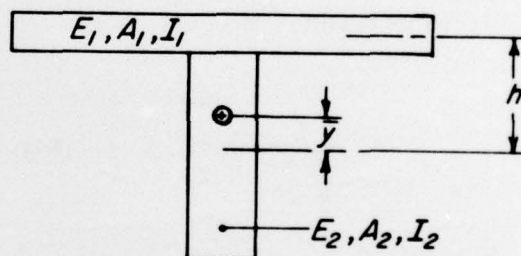
Figure B-1. -- Beam schematic for the "transformed area" method.   (M 144 651)

Appendix C. -- Sample Calculations for Theoretical Floor and T-beam Deflections

FPL Floors

At 50 pounds per square foot distributed load, deflection of the FPL floors (table 5) can be computed by:

$$\Delta = \frac{5}{384}\,\frac{wL^4}{EI} = \frac{5}{384}\,\frac{\left(\frac{50}{144}\times 16\right)(144)^4}{EI}$$

$$= \frac{31.104\times 10^6}{EI}$$

For floors G-2 and N-2, the 2-by-8 joists had an average E of 2.20×10^6 pounds per square inch, giving EI = 104.80×10^6 pounds per square inch and EA = 23.93×10^6 pounds. By test, the 5/8-inch sheathing had an EA of 5.00×10^6 pounds. From Eq. (8):

$$EI_R = 104.80 + \frac{(23.93)\,(5.00)}{28.93}(3.9375)^2$$

$$= 104.80 + 64.12$$
$$= 168.92\times 10^6$$

$$\Delta_R = \frac{31.104}{168.92} = 0.184\ inch$$

This is the theoretical deflection for G-2, which used a rigid adhesive and continuous sheathing.

Floor N-2 used 8d nails at 6 inches along the panel edges and 8 inches over intermediate joists. For each full sheet of plywood, the average nail spacing was 7.43 inch (five intermediate rows at 8 in. and two edge rows at 6 in.). From table 2:

$$S = \frac{9400}{7.43} = 1265\ lb/in.$$

Gaps were left between the plywood sheets, giving L' = 48 inches.

Therefore, from Eqs. (6), (11), and (5), theoretical deflection for N-2 can be computed:

$$(L'\alpha)^2 = \frac{(48)^2(3.9375)^2(1265)}{(168.92 - 104.80)\times 10^6}\left(\frac{168.92}{104.80}\right) = 1.136$$

$$f_\Delta = \frac{10}{1.136 + 10} = 0.898$$

$$\Delta = 0.184\left[1 + 0.898\left(\frac{168.92}{104.80} - 1\right)\right]$$

$$= 0.285\ inch$$

CSU T-beams

Beam T12 of the CSU T-beams (table 5) consisted of two 2-by-8 joists at 16 inches on centers and 3/4-inch plywood sheathing. Fasteners were 8d common nails at 6 inches. The beam was tested under concentrated midspan load over a span of 12 feet. The computations for joist 1 are based on reported properties (3):

$$E(joist) = 1.249\ x\ 10^6\ lb/in.^2$$
$$E(plywood\ in\ bending) = 0.5581\ x\ 10^6$$
$$E(plywood\ under\ axial\ load) = 0.9178\ x\ 10^6$$

This gives the following values:

	EA	EI
Joist	$13.58\ x\ 10^6$	$59.50\ x\ 10^6$
Sheathing	11.01	0.31
Total	24.59	59.81

From Eq. (3) and table 2:

$$S = \frac{P/\delta}{s} = \frac{10,200}{6} = 1700$$

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Equation (8):

$$EI_R = 59.81 + \frac{(13.58)(11.01)}{24.59}(4)^2 = 59.81 + 97.29$$
$$= 157.10 \times 10^6$$

At 375 lb:

$$\triangle_R = \frac{PL^3}{48EI_R} = \frac{(375)(144)^3}{(48)(157.10 \times 10^6)} = 0.148 \ inch$$

For one gap (L ' = 72 in.):
Equation (6):

$$(L'\alpha)^2 = \frac{(72)^2(4)^2(1700)}{97.29 \times 10^6}\left(\frac{157.10}{59.81}\right) = 3.807$$

Equation (11):

$$f_\triangle = \frac{10}{3.807 + 10} = 0.724$$

Equation (5):

$$\triangle = 0.148\left[1 + 0.724\left(\frac{157.10}{59.81} - 1\right)\right] = 0.323 \ inch$$

Similarly, for five gaps (L' = 24 in.):

$$(L'\alpha)^2 = 0.423$$
$$f_\triangle = 0.959$$
$$\triangle = 0.379 \ inch$$

NAHB Floors

Floor No. 4 of the NAHB series (table 6) consisted of 2-by-6 joists and 3/4-inch plywood sheathing with gaps every 4 feet. The following values were reported (5):

$$EI_U = 37.46 \times 10^6 \ lb\text{-}in.^2$$
$$EI_R = 92.49$$
$$EI \ (test) = 76.31$$

Therefore

$$\frac{EI_R}{EI_U} = 2.469$$

and Eq. (6) can be written as:

$$(L'\alpha)^2 = \frac{(48)^2(3.125)^2 S}{55.03 \times 10^6}(2.469) = 1.0095 \times 10^{-3} S$$

First, assume S = 25,000 lb/in.²

$$(L'\alpha)^2 = 25.24$$

Equation (11):

$$f_\triangle = \frac{10}{25.24 + 10} = 0.284$$

Equation (9):

$$EI = \frac{92.49}{1 + 0.284(1.469)} = 65.28 \times 10^6$$

Similarly,

for S = 50,000:	and for S = 100,000:
$(L'\alpha)^2 = 50.47$	$(L'\alpha)^2 = 100.95$
$f_\triangle = 0.165$	$f_\triangle = 0.090$
$EI = 74.41 \times 10^6$	$EI = 81.68 \times 10^6$

U.S. Forest Products Laboratory.

Method for predicting the stiffness of wood-joist floor systems with partial composite action, by William J. McCutcheon. Madison, Wis. FPL, 1977. 17 p. (USDA For. Serv. Res. Pap. FPL 289).

Using methods for computing stiffness of composite beams, and for predicting load-slip characteristics of individual mechanical fasteners, complex computational procedures were reworked and combined. Open gaps in sheathing were considered with a simple modification of the basic method.
Keywords: Wood floors, T-beam, Composite action, Stiffness, Deflection, Concentrated load, Uniform load, Nail-glued, Nailed.